Below is a summary of the methods called within the analyze\_results method of the EnhancedBayesianChoiceAnalyzer class, along with their functionality and interpretations of the results, including utilities with uncertainty. The analyze\_results method orchestrates the analysis of the fitted Bayesian choice model by invoking several internal methods to evaluate utilities, feature importance, willingness-to-pay (WTP), price elasticity, and market segments.

Overview of analyze\_results

The analyze\_results method is the central function for post-model-fitting analysis. It performs the following tasks by calling five internal methods:

1. Plots feature utilities with uncertainty.

2. Calculates and visualizes feature importance.

3. Analyzes willingness-to-pay (WTP) with uncertainty intervals.

4. Evaluates price elasticity across price levels.

5. Compares preferences across market segments.

6. Generates a Chart.js bar chart for WTP visualization.

Each method produces outputs (e.g., plots, data frames, or dictionaries) that provide insights into consumer preferences, feature impacts, and market dynamics. Below, I summarize each method’s purpose, implementation, and interpretation of results, focusing on utilities and their uncertainty.

1. \_plot\_utilities\_with\_uncertainty

# Purpose

This method visualizes the estimated utilities (coefficients, or betas) for each feature in the choice model, along with their 95% credible intervals, to show the impact of each feature on choice probability.

# Implementation

- Input: Uses the posterior distribution of betas from the model trace (self.trace).

- Process:

- Calls az.plot\_forest from the ArviZ library to create a forest plot.

- The plot displays the mean utility (point estimate) and 95% highest density interval (HDI) for each feature listed in self.X\_cols.

- Feature names are formatted using self.rename\_dict and wrapped for readability using textwrap.

- The plot is customized with feature names on the y-axis and utility values on the x-axis.

- Output: Saves a forest plot as utilities\_with\_uncertainty.png.

# Interpretation of Results

- Utilities: The betas represent the utility (or preference weight) of each feature in the choice model. A positive beta indicates that the feature increases the likelihood of a profile being chosen, while a negative beta decreases it. For example, a negative beta for Price (as expected) suggests that higher prices reduce choice probability.

- Uncertainty: The 95% HDI provides a range within which the true utility is likely to lie with 95% probability. Narrow HDIs indicate precise estimates, while wide HDIs suggest greater uncertainty, possibly due to limited data or variability in preferences.

- Practical Insight: Features with large positive utilities (e.g., advanced features like Adv\_Feat\_Safety) are highly valued by respondents. If the HDI for a feature includes zero, its effect may not be statistically significant, warranting caution in decision-making.

2. \_calculate\_feature\_importance

# Purpose

This method quantifies and visualizes the relative importance of each feature based on the range of utilities it contributes to the choice decision.

# Implementation

- Input: Uses the posterior distribution of betas extracted from self.trace.

- Process:

- For Price, importance is calculated as the absolute utility (|beta|) multiplied by the normalized price range (max - min price divided by 1000).

- For categorical features (e.g., SP or AF levels), importance is the difference between the maximum and minimum utilities across their levels.

- Importance values are normalized by dividing by the sum of all feature importances to yield relative contributions (summing to 1).

- Results are stored in self.feature\_importance as a dictionary.

- A bar plot is created using seaborn.barplot, showing features sorted by importance.

- Output: Saves a bar plot as feature\_importance.png and returns self.feature\_importance.

# Interpretation of Results

- Feature Importance: The normalized importance scores indicate the relative contribution of each feature to choice decisions. A higher score means the feature has a larger impact. For example, if Adv\_Feat\_Safety has a high importance score, it strongly influences choices compared to less impactful features like Adv\_Feat\_ElecLife.

- Uncertainty: Since importance is derived from posterior betas, uncertainty is indirectly reflected in the variability of beta estimates. However, the method does not explicitly plot uncertainty for importance scores.

- Practical Insight: Features with high importance are key drivers of consumer preference and should be prioritized in product design or marketing. For instance, if Price has high importance, pricing strategies will significantly affect market share.

3. \_analyze\_willingness\_to\_pay

# Purpose

This method estimates the willingness-to-pay (WTP) for each feature (except Price) and quantifies uncertainty in these estimates. WTP represents how much additional price respondents are willing to pay for a feature.

# Implementation

- Input: Uses the posterior distribution of WTP variables (wtp\_{feature}) from self.trace, calculated as -beta\_feature / beta\_price (scaled by 1000 to convert to currency units).

- Process:

- For each non-price feature, extracts WTP values and computes:

- Mean WTP.

- 95% HDI for uncertainty.

- Median WTP.

- Standard deviation.

- Creates an error bar plot with mean WTP as points, 95% HDI as error bars, and median WTP as red ‘x’ markers.

- Adds reference lines for zero WTP and the median price (positive and negative) from the original profiles.

- Results are stored in a pandas DataFrame (wtp\_df).

- Output: Saves an error bar plot as wtp\_analysis\_enhanced.png and exports wtp\_df to wtp\_results.xlsx. Returns wtp\_df.

# Interpretation of Results

- WTP: The mean WTP indicates the additional price (in currency units) respondents are willing to pay for a feature. For example, a WTP of 500 for Adv\_Feat\_Safety means respondents value this feature at an additional 500 units over a baseline profile.

- Uncertainty: The 95% HDI shows the range of plausible WTP values. A wide HDI (e.g., [200, 800]) indicates uncertainty, while a narrow HDI (e.g., [450, 550]) suggests a precise estimate. If the HDI includes zero, the WTP may not be significant, implying the feature’s value is uncertain.

- Median vs. Mean: The median WTP is less sensitive to outliers in the posterior distribution. A large difference between mean and median suggests a skewed WTP distribution, which may reflect heterogeneous preferences.

- Practical Insight: Features with high WTP and narrow HDIs are strong candidates for premium offerings. For instance, if Adv\_Feat\_ModbusEth has a high WTP with low uncertainty, it could justify its inclusion in a high-priced product. Conversely, features with low or negative WTP may not support price premiums.

4. \_analyze\_price\_elasticity

# Purpose

This method analyzes how sensitive choice probabilities are to price changes, providing insights into how price affects demand.

# Implementation

- Input: Uses the posterior distribution of price\_elasticity from self.trace, calculated as beta\_price \* price \* (1 - choice\_prob).

- Process:

- Creates a posterior plot of the overall elasticity distribution of using az.plot\_posterior to show its distribution and 95% HDI.

- Segments elasticity by price levels using pd.qcut to divide prices into four quartiles.

- Computes mean and standard deviation of elasticity for each price quartile.

- Creates a bar chart with mean elasticity per quartile, with error bars for standard deviation.

- Output: Saves a posterior distribution plot as price\_elasticity.png and a bar chart as elasticity\_by\_price\_level.png. Returns a DataFrame with elasticity statistics by price level.

# Interpretation

- Price Elasticity: Elasticity measures the percentage change in choice probability for a percentage change in price. A value of -1 means a 1% price increase reduces choice probability by 1%. Values less than -1 indicate elastic demand (sensitive to price), while values greater than -1 indicate inelastic demand.

- Uncertainty: The 95% HDI in the posterior plot shows uncertainty in the overall elasticity estimate. The standard deviation in the bar chart reflects variability within price quartiles.

- Price Levels: Higher elasticity in higher price quartiles suggests that consumers are more price-sensitive at premium price points, which is typical in choice models.

- Practical Insight: If elasticity is highly elastic at high prices, price increases may significantly reduce market share, suggesting a need for competitive pricing or value-added features to maintain demand.

5. \_analyze\_market\_segments

# Purpose

This method compares feature preferences across market segments (groups defined in self.groups), highlighting differences in utilities.

# Implementation

- Input: Uses the posterior distribution of betas from self.trace.

- Process:

- For each feature, computes the mean utility and 95% HDI.

- Stores results in a dictionary (comparisons).

- Creates a separate error bar plot for each feature, showing the mean utility and HDI, with a vertical line at zero for reference.

- Output: Saves individual plots as comparison\_{feature}.png for each feature. Returns the comparisons dictionary.

# Interpretation of Results

- Segment Preferences: The mean utility indicates the preference strength for a feature within a segment. Differences in mean utilities across segments (if groups are defined) reveal segment-specific preferences.

- Uncertainty: The 95% HDI shows the uncertainty in utility estimates. Overlapping HDIs across segments suggest no significant preference differences, while non-overlapping HDIs indicate distinct segment priorities.

- Practical Insight: If Adv\_Feat\_Health has a high utility in one segment but low in another, targeted marketing can emphasize this feature for the high-preference segment. Wide HDIs may indicate a need for more data to confirm segment differences.

6. Chart.js Bar Chart for WTP

# Purpose

This method generates an interactive Chart.js bar chart to visualize WTP estimates with 95% error bars, enhancing the WTP analysis for presentation purposes.

# Implementation

- Input: Uses the WTP results from \_analyze\_willingness\_to\_pay.

- Process:

- Extracts feature names (renamed via self.rename\_dict), mean WTP, and 95% HDI bounds.

- Computes error bars as the difference between mean WTP and HDI bounds.

- Creates a Chart.js configuration for a horizontal bar chart:

- Labels: Feature names.

- Data: Mean WTP values.

- Error Bars: HDI bounds as error bars.

- Styling: Blue bars with white text, transparent background for compatibility with dark/light themes.

- Output: Logs the Chart.js configuration (not saved as a file).

# Interpretation of Results Below is a summary of the methods within the analyze\_results method of the provided EnhancedBayesianChoiceAnalyzer class, along with interpretations of the results, including utilities with uncertainty.

Overview of analyze\_results

The analyze\_results method orchestrates the analysis of the fitted Bayesian choice model by calling several helper methods to generate insights into utilities, feature importance, willingness-to-pay (WTP), price elasticity, and market segments. It also produces a Chart.js bar chart for WTP visualization. Each helper method generates specific outputs (e.g., plots, data frames) and provides interpretations of the model’s results.

Methods in analyze\_results

1. \_plot\_utilities\_with\_uncertainty

- Purpose: Visualizes the utility coefficients (betas) for each feature with 95% credible intervals.

- Implementation:

- Uses az.plot\_forest from ArviZ to create a forest plot of the posterior distribution of betas.

- Features are labeled using rename\_dict for clarity, with text wrapped for readability.

- Saves the plot as utilities\_with\_uncertainty.png.

- Interpretation:

- The plot shows the estimated utility (preference) for each feature, with error bars representing the 95% highest density interval (HDI).

- A positive utility indicates that the feature increases the likelihood of a profile being chosen, while a negative utility (e.g., for Price) suggests a deterrent effect.

- The width of the HDI reflects uncertainty: wider intervals indicate less certainty in the utility estimate, possibly due to limited data or high variability in preferences.

- Example: If Adv\_Feat\_Safety has a mean utility of 0.5 with a narrow HDI (e.g., [0.4, 0.6]), it strongly and reliably increases choice probability. If Price has a negative utility (e.g., -0.3) with a wide HDI (e.g., [-0.5, -0.1]), the negative effect is less certain.

2. \_calculate\_feature\_importance

- Purpose: Quantifies and visualizes the relative importance of features based on their utility ranges.

- Implementation:

- For Price, importance is calculated as the absolute utility multiplied by the price range (normalized to thousands).

- For categorical features (e.g., SP, AF), importance is the difference between the maximum and minimum utilities of their levels.

- Normalizes importance scores to sum to 1 and creates a bar plot saved as feature\_importance.png.

- Returns a dictionary of feature importance scores.

- Interpretation:

- Features with higher importance scores have a greater impact on choice decisions. For example, if Price has an importance of 0.4, it influences 40% of the decision variance relative to other features.

- The plot ranks features, helping identify which attributes (e.g., Adv\_Feat\_Safety vs. Size\_Perf\_High) drive preferences most.

- Uncertainty is indirectly captured through the utility estimates’ posterior distributions, but the method focuses on mean importance. High variance in utility estimates could lead to less reliable importance scores.

3. \_analyze\_willingness\_to\_pay

- Purpose: Estimates WTP for each feature (excluding Price) with uncertainty intervals and visualizes results.

- Implementation:

- Extracts WTP values from the posterior (wtp\_{feature}), calculated as -β\_feature / β\_Price \* 1000.

- Computes mean, median, standard deviation, and 95% HDI for each feature’s WTP.

- Creates an error bar plot with mean WTP, 95% HDI, and median (marked with an ‘x’), including reference lines for median price and zero WTP.

- Saves the plot as wtp\_analysis\_enhanced.png and results to wtp\_results.xlsx.

- Returns a DataFrame with WTP statistics.

- Interpretation:

- WTP represents how much respondents are willing to pay for a feature relative to the baseline (e.g., the dropped level of a categorical feature).

- Example: If Adv\_Feat\_Health has a mean WTP of 5000 (currency units) with HDI [3000, 7000], respondents are willing to pay approximately 5000 units for this feature, with 95% confidence that the true WTP lies within 3000–7000.

- Negative WTP values (if any) suggest respondents would need a discount to accept a feature.

- The HDI width indicates uncertainty: a wide HDI (e.g., [1000, 9000]) suggests high variability in preferences or model sensitivity to noise, reducing confidence in the estimate.

- The median price reference lines contextualize WTP against typical product costs, aiding practical decision-making (e.g., is the WTP feasible given production costs?).

4. \_analyze\_price\_elasticity

- Purpose: Analyzes price elasticity of demand across price levels.

- Implementation:

- Extracts the posterior distribution of price\_elasticity, calculated as β\_Price \* Price \* (1 - choice\_prob).

- Creates a posterior distribution plot saved as price\_elasticity\_distribution.png.

- Groups price levels into quartiles and computes mean and standard deviation of elasticity for each.

- Plots a bar chart of elasticity by price level with error bars, saved as elasticity\_by\_price\_level.png.

- Returns a DataFrame with elasticity statistics.

- Interpretation:

- Price elasticity measures how sensitive demand is to price changes. A value of -1 means a 1% price increase reduces demand by 1%.

- Example: If elasticity is -0.8 for the lowest price quartile (HDI [-1.0, -0.6]), demand is relatively inelastic (less sensitive to price changes), suggesting room for price increases without significant demand loss.

- Higher (more negative) elasticity in higher price quartiles indicates greater sensitivity, where price increases could significantly reduce demand.

- The standard deviation and HDI reflect uncertainty: high variability suggests less reliable estimates, possibly due to heterogeneous preferences or model fit issues.

5. \_analyze\_market\_segments

- Purpose: Compares feature preferences across market segments (based on the group variable).

- Implementation:

- Extracts betas and computes mean and 95% HDI for each feature.

- Creates individual error bar plots for each feature’s utility, saved as comparison\_{feature}.png.

- Returns a dictionary with utility statistics for each feature.

- Interpretation:

- This method examines whether preferences differ across respondent groups (e.g., different customer segments).

- Example: If Adv\_Feat\_ModbusEth has a utility of 0.3 (HDI [0.1, 0.5]) for one group and 0.1 (HDI [-0.1, 0.3]) for another, the first group values this feature more, though overlapping HDIs suggest the difference may not be statistically significant.

- The plots highlight features with distinct preferences across segments, guiding targeted marketing or product design.

- Uncertainty in utilities (wide HDIs) may obscure segment differences, indicating a need for more data or refined segmentation.

6. Chart.js WTP Visualization (within analyze\_results)

- Purpose: Generates an interactive bar chart for WTP with error bars for 95% HDI.

- Implementation:

- Uses WTP results to create a horizontal bar chart with mean WTP values and error bars for HDI bounds.

- Configures Chart.js with appropriate colors, labels, and scales for clarity.

- Logs the generation but does not save the chart (assumed to be displayed in a UI).

- Interpretation:

- The chart visually summarizes WTP, making it easy to compare features’ economic value.

- Error bars show uncertainty: longer bars indicate less precise WTP estimates.

- Example: A bar for Adv\_Feat\_Safety at 6000 with short error bars suggests high confidence in a high WTP, while a bar for Size\_Perf\_High at 2000 with long error bars indicates uncertainty, reducing reliability for pricing decisions.

General Interpretation of Results

- Utilities with Uncertainty:

- Utilities (betas) quantify the impact of each feature on choice probability. Positive utilities increase preference, while negative utilities (e.g., Price) decrease it.

- The 95% HDI provides a range of plausible utility values, reflecting estimation uncertainty. Narrow HDIs indicate robust estimates, while wide HDIs suggest variability or insufficient data.

- Example: A utility of 0.4 for Adv\_Feat\_Safety with HDI [0.3, 0.5] is more reliable than a utility of 0.2 for Adv\_Feat\_TempMon with HDI [-0.1, 0.5].

- Feature Importance:

- Importance scores highlight which features drive choices most. High importance for Price suggests cost sensitivity, while high importance for advanced features (e.g., Adv\_Feat\_Health) indicates preference for functionality.

- Uncertainty in utilities propagates to importance scores, so features with wide HDI utilities may have less reliable importance estimates.

- Willingness-to-Pay:

- WTP translates utilities into monetary terms, guiding pricing strategies. High WTP for features like Adv\_Feat\_Safety suggests premium pricing potential, while low WTP for Size\_Perf\_High may indicate limited value.

- Uncertainty (HDI) is critical: wide HDIs suggest caution in pricing decisions, as the true WTP may vary significantly.

- Example: A WTP of 5000 for Adv\_Feat\_Health with HDI [3000, 7000] supports a price premium, but a WTP of 1000 for Adv\_Feat\_TempMon with HDI [-500, 2500] is too uncertain for confident pricing.

- Price Elasticity:

- Elasticity informs how price changes affect demand. Inelastic demand (elasticity closer to 0) at lower prices suggests flexibility to raise prices, while elastic demand at higher prices warns against price hikes.

- Uncertainty in elasticity estimates (via HDI or standard deviation) indicates the reliability of these insights. High uncertainty may require further data collection.

- Market Segments:

- Segment analysis reveals preference heterogeneity, enabling targeted strategies. Significant utility differences (non-overlapping HDIs) across groups highlight opportunities for segment-specific products.

- Uncertainty in segment utilities can obscure differences, suggesting a need for refined group definitions or more respondents.

Practical Implications

- Product Design: Prioritize features with high utility and WTP (e.g., Adv\_Feat\_Safety, Adv\_Feat\_Health) in product development.

- Pricing Strategy: Use WTP and elasticity to set prices. Features with high, reliable WTP justify premiums, while elastic demand at high prices suggests discounts or value-focused marketing.

- Marketing: Target segments with distinct preferences (e.g., tech-savvy groups valuing Adv\_Feat\_ModbusEth) with tailored campaigns.

- Uncertainty Management: Wide HDIs in utilities, WTP, or elasticity indicate areas needing more data or refined modeling to reduce risk in decision-making.

Limitations and Considerations

- Model Assumptions: The non-hierarchical model assumes homogeneous preferences, which may oversimplify real-world heterogeneity. A hierarchical model could capture individual differences better.

- Uncertainty: High uncertainty (wide HDIs) in some estimates reduces confidence in results, particularly for features with low importance or sparse data.

- Data Quality: Errors in profiles, choices, or groups (e.g., typos in AF) could bias results, though the code includes validation steps.

- ADVI Approximation: Using ADVI (variational inference) instead of MCMC may lead to less accurate posterior estimates, especially for complex distributions.

This summary provides a comprehensive overview of the analyze\_results method’s components and their interpretations, emphasizing the role of uncertainty in decision-making. Let me know if you need further details or assistance with specific aspects of the code or results!